

CHAPTER 21 (Odd)

1. a. $M = k\sqrt{L_p L_s} \Rightarrow L_s = \frac{M^2}{L_p k^2} = \frac{(80 \text{ mH})^2}{(50 \text{ mH})(0.8)^2} = \mathbf{0.2 \text{ H}}$
- b. $e_p = N_p \frac{d\phi_p}{dt} = (20)(0.08 \text{ Wb/s}) = \mathbf{1.6 \text{ V}}$
 $e_s = kN_s \frac{d\phi_p}{dt} = (0.8)(80 \text{ t})(0.08 \text{ Wb/s}) = \mathbf{5.12 \text{ V}}$
- c. $e_p = L_p \frac{di_p}{dt} = (50 \text{ mH})(0.03 \times 10^3 \text{ A/s}) = \mathbf{15 \text{ V}}$
 $e_s = M \frac{di_p}{dt} = (80 \text{ mH})(0.03 \times 10^3 \text{ A/s}) = \mathbf{24 \text{ V}}$
3. a. $L_s = \frac{M^2}{L_p k^2} = \frac{(80 \text{ mH})^2}{(50 \text{ mH})(0.9)^2} = \mathbf{158.02 \text{ mH}}$
- b. $e_p = N_p \frac{d\phi_p}{dt} = (300 \text{ t})(0.08 \text{ Wb/s}) = \mathbf{24 \text{ V}}$
 $e_s = kN_s \frac{d\phi_p}{dt} = (0.9)(25 \text{ t})(0.08 \text{ Wb/s}) = \mathbf{1.8 \text{ V}}$
- c. e_p and e_s the same as problem 1: $e_p = \mathbf{15 \text{ V}}$, $e_s = \mathbf{24 \text{ V}}$
5. a. $E_s = \frac{N_s}{N_p} E_p = \frac{30 \text{ t}}{240 \text{ t}} (25 \text{ V}) = \mathbf{3.125 \text{ V}}$
- b. $\Phi_{m(\max)} = \frac{E_p}{4.44 f N_p} = \frac{25 \text{ V}}{(4.44)(60 \text{ Hz})(240 \text{ t})} = \mathbf{391.02 \mu\text{Wb}}$
7. $f = \frac{E_p}{(4.44) N_p \Phi_{m(\max)}} = \frac{25 \text{ V}}{(4.44)(8 \text{ t})(12.5 \text{ mWb})} = \mathbf{56.31 \text{ Hz}}$
9. $Z_p = \frac{V_g}{I_p} = \frac{1600 \text{ V}}{4 \text{ A}} = \mathbf{400 \Omega}$
11. $I_L = I_s = \frac{V_L}{Z_L} = \frac{240 \text{ V}}{20 \Omega} = 12 \text{ A}$
 $\frac{I_s}{I_p} = a = \frac{N_p}{N_s} \Rightarrow \frac{12 \text{ A}}{0.05 \text{ A}} = \frac{N_p}{50}$
 $N_p = \frac{50(12)}{0.05} = \mathbf{12,000 \text{ turns}}$

13. a. $Z_p = a^2 Z_L \Rightarrow a = \sqrt{\frac{Z_p}{Z_L}}$
 $Z_p = \frac{V_p}{I_p} = \frac{10 \text{ V}}{20 \text{ V}/72 \Omega} = 36 \Omega$
 $a = \sqrt{\frac{36 \Omega}{4 \Omega}} = 3$
- b. $\frac{V_s}{V_p} = \frac{N_s}{N_p} = \frac{1}{3} \Rightarrow V_s = \frac{1}{3} V_p = \frac{1}{3}(10 \text{ V}) = 3\frac{1}{3} \text{ V}$
 $P = \frac{V_s^2}{Z_s} = \frac{(3.33 \text{ V})^2}{4 \Omega} = 2.78 \text{ W}$
15. a. $a = \frac{N_p}{N_s} = \frac{4t}{1t} = 4$
 $R_e = R_p + a^2 R_s = 4 \Omega + (4)^2 1 \Omega = 20 \Omega$
 $X_e = X_p + a^2 X_s = 8 \Omega + (4)^2 2 \Omega = 40 \Omega$
 $Z_p = Z_{R_e} + Z_{X_e} + a^2 Z_{X_L} = 20 \Omega + j40 \Omega + j(4)^2 20 \Omega$
 $= 20 \Omega + j40 \Omega + j320 \Omega = 20 \Omega + j360 \Omega = 360.56 \Omega \angle 86.82^\circ$
- b. $I_p = \frac{V_g}{Z_p} = \frac{120 \text{ V} \angle 0^\circ}{360.56 \Omega \angle 86.82^\circ} = 332.82 \text{ mA} \angle -86.82^\circ$
- c. $V_{R_e} = (I \angle \theta)(R_e \angle 0^\circ) = (332.82 \text{ mA} \angle -86.82^\circ)(20 \Omega \angle 0^\circ)$
 $= 6.656 \text{ V} \angle -86.82^\circ$
 $V_{X_e} = (I \angle \theta)(X_e \angle 90^\circ) = (332.82 \text{ mA} \angle -86.82^\circ)(40 \Omega \angle 90^\circ)$
 $= 13.313 \text{ V} \angle 3.18^\circ$
 $V_{X_L} = I(a^2 Z_{X_L}) = (332.82 \text{ mA} \angle -86.82^\circ)(320 \Omega \angle 90^\circ)$
 $= 106.50 \text{ V} \angle 3.18^\circ$
17. —
19. $L_{T(+) } = L_1 + L_2 + 2M_{12}$
 $M_{12} = k\sqrt{L_1 L_2} = (0.8)\sqrt{(200 \text{ mH})(600 \text{ mH})} = 277 \text{ mH}$
 $L_{T(+) } = 200 \text{ mH} + 600 \text{ mH} + 2(277 \text{ mH}) = 1.354 \text{ H}$
21. $E_1 - I_1[Z_{R_1} + Z_{L_1}] - I_2[Z_m] = 0$
 $I_2[Z_{L_2} + Z_{R_L}] + I_1[Z_m] = 0$

 $I_1(Z_{R_1} + Z_{L_1}) + I_2(Z_m) = E_1$
 $I_1(Z_m) + I_2(Z_{L_2} + Z_{R_L}) = 0 \quad X_m = -\omega M \angle 90^\circ$

23. a. $a = \frac{N_p}{N_s} = \frac{V_p}{V_s} = \frac{2400 \text{ V}}{120 \text{ V}} = 20$

b. $10,000 \text{ VA} = V_s I_s \Rightarrow I_s = \frac{10,000 \text{ VA}}{V_s} = \frac{10,000 \text{ VA}}{120 \text{ V}} = 83.33 \text{ A}$

c. $I_p = \frac{10,000 \text{ VA}}{V_p} = \frac{10,000 \text{ VA}}{2400 \text{ V}} = 4.167 \text{ A}$

d. $a = \frac{V_p}{V_s} = \frac{120 \text{ V}}{2400 \text{ V}} = 0.05 = \frac{1}{20}$
 $I_s = \frac{10,000 \text{ VA}}{2400 \text{ V}} = 4.167 \text{ A}, I_p = 83.33 \text{ A}$

25. a. $E_s = \frac{N_s}{N_p} E_p$
 $= \frac{25 \text{ t}}{100 \text{ t}} (100 \text{ V} \angle 0^\circ) = 25 \text{ V} \angle 0^\circ = V_L$
 $I_s = \frac{E_s}{Z_L} = \frac{25 \text{ V} \angle 0^\circ}{5 \Omega \angle 0^\circ} = 5 \text{ A} \angle 0^\circ = I_L$

b. $Z_i = a^2 Z_L = \left(\frac{N_p}{N_s} \right)^2 Z_L = \left(\frac{100 \text{ t}}{25 \text{ t}} \right)^2 5 \Omega \angle 0^\circ = (4)^2 5 \Omega \angle 0^\circ = 80 \Omega \angle 0^\circ$

c. $Z_{1/2} = \frac{1}{4} Z_i = \frac{1}{4} (80 \Omega \angle 0^\circ) = 20 \Omega \angle 0^\circ$

27. a. $E_2 = \frac{N_2}{N_1} E_1 = \left(\frac{40 \text{ t}}{120 \text{ t}} \right) (120 \text{ V} \angle 60^\circ) = 40 \text{ V} \angle 60^\circ$
 $I_2 = \frac{E_2}{Z_2} = \frac{40 \text{ V} \angle 60^\circ}{12 \Omega \angle 0^\circ} = 3.33 \text{ A} \angle 60^\circ$
 $E_3 = \frac{N_3}{N_1} E_1 = \left(\frac{30 \text{ t}}{120 \text{ t}} \right) (120 \text{ V} \angle 60^\circ) = 30 \text{ V} \angle 60^\circ$
 $I_3 = \frac{E_3}{Z_3} = \frac{30 \text{ V} \angle 60^\circ}{10 \Omega \angle 0^\circ} = 3 \text{ A} \angle 60^\circ$

b. $\frac{1}{R_1} = \frac{1}{(N_1/N_2)^2 R_2} + \frac{1}{(N_1/N_3)^2 R_3}$
 $= \frac{1}{(120 \text{ t}/40 \text{ t})^2 12 \Omega} + \frac{1}{(120 \text{ t}/30 \text{ t})^2 10 \Omega}$
 $\frac{1}{R_1} = \frac{1}{108 \Omega} + \frac{1}{160 \Omega} = 0.0155 \text{ S}$
 $R_1 = \frac{1}{0.0155 \text{ S}} = 64.52 \Omega$

$$\begin{aligned}
29. \quad & \mathbf{E}_1 - \mathbf{I}_1 \mathbf{Z}_1 - \mathbf{I}_1 \mathbf{Z}_{L_1} - \mathbf{I}_2(-\mathbf{Z}_{M_{12}}) - \mathbf{I}_3(+\mathbf{Z}_{M_{13}}) = 0 \\
\text{or} \quad & \mathbf{E}_1 - \mathbf{I}_1[\mathbf{Z}_1 + \mathbf{Z}_{L_1}] + \mathbf{I}_2 \mathbf{Z}_{M_{12}} - \mathbf{I}_3 \mathbf{Z}_{M_{13}} = 0 \\
\hline
& -\mathbf{I}_2(\mathbf{Z}_2 + \mathbf{Z}_3 + \mathbf{Z}_{L_2}) + \mathbf{I}_3 \mathbf{Z}_2 - \mathbf{I}_1(-\mathbf{Z}_{M_{12}}) = 0 \\
\text{or} \quad & -\mathbf{I}_2(\mathbf{Z}_2 + \mathbf{Z}_3 + \mathbf{Z}_{L_2}) + \mathbf{I}_3 \mathbf{Z}_2 + \mathbf{I}_1 \mathbf{Z}_{M_{12}} = 0 \\
\hline
& -\mathbf{I}_3(\mathbf{Z}_2 + \mathbf{Z}_4 + \mathbf{Z}_{L_3}) + \mathbf{I}_2 \mathbf{Z}_2 - \mathbf{I}_1(+\mathbf{Z}_{M_{13}}) = 0 \\
\text{or} \quad & -\mathbf{I}_3(\mathbf{Z}_2 + \mathbf{Z}_4 + \mathbf{Z}_{L_3}) + \mathbf{I}_2 \mathbf{Z}_2 - \mathbf{I}_1 \mathbf{Z}_{M_{13}} = 0 \\
\hline
\therefore \quad & \begin{aligned}
& [\mathbf{Z}_1 + \mathbf{Z}_{L_1}] \mathbf{I}_1 - \mathbf{Z}_{M_{12}} \mathbf{I}_2 + \mathbf{Z}_{M_{13}} \mathbf{I}_3 = \mathbf{E}_1 \\
& \mathbf{Z}_{M_{12}} \mathbf{I}_1 - [\mathbf{Z}_2 + \mathbf{Z}_3 + \mathbf{Z}_{L_2}] \mathbf{I}_2 + \mathbf{Z}_2 \mathbf{I}_3 = 0 \\
& \mathbf{Z}_{M_{13}} \mathbf{I}_1 \mathbf{Z}_2 \mathbf{I}_2 + [\mathbf{Z}_2 + \mathbf{Z}_4 + \mathbf{Z}_{L_3}] \mathbf{I}_3 = 0
\end{aligned}
\end{aligned}$$

CHAPTER 21 (Even)

2. a. $k = 1$

$$(a) \quad L_s = \frac{M^2}{L_p k^2} = \frac{(80 \text{ mH})^2}{(50 \text{ mH})(1)^2} = 128 \text{ mH}$$

$$(b) \quad e_p = 1.6 \text{ V}, e_s = kN_s \frac{d\phi_p}{dt} = (1)(80 \text{ t})(0.08 \text{ Wb/s}) = 6.4 \text{ V}$$

$$(c) \quad e_p = 15 \text{ V}, e_s = 24 \text{ V}$$

b. $k = 0.2$

$$(a) \quad L_s = \frac{M^2}{L_p k^2} = \frac{(80 \text{ mH})^2}{(50 \text{ mH})(0.2)^2} = 3.2 \text{ H}$$

$$(b) \quad e_p = 1.6 \text{ V}, e_s = kN_s \frac{d\phi_p}{dt} = (0.2)(80 \text{ t})(0.08 \text{ Wb/s}) = 1.28 \text{ V}$$

$$(c) \quad e_p = 15 \text{ V}, e_s = 24 \text{ V}$$

4. a. $E_s = \frac{N_s}{N_p} E_p = \frac{64 \text{ t}}{8 \text{ t}} (25 \text{ V}) = 200 \text{ V}$

b. $\Phi_{\max} = \frac{E_p}{4.44fN_p} = \frac{25 \text{ V}}{4.44(60 \text{ Hz})(8 \text{ t})} = 11.73 \text{ mWb}$

6. $E_p = \frac{N_p}{N_s} E_s = \frac{60 \text{ t}}{720 \text{ t}} (240 \text{ V}) = 20 \text{ V}$

8. a. $I_L = aI_p = \left[\frac{1}{5} \right] (2 \text{ A}) = 0.4 \text{ A}$
 $V_L = I_L Z_L = \left[\frac{2}{5} \text{ A} \right] (2 \Omega) = 0.8 \text{ V}$

b. $Z_{\text{in}} = a^2 Z_L = \left[\frac{1}{5} \right]^2 2 \Omega = 0.08 \Omega$

10. $V_g = aV_L = \left[\frac{1}{4} \right] (1200 \text{ V}) = 300 \text{ V}$

$$I_p = \frac{V_g}{Z_i} = \frac{300 \text{ V}}{4 \Omega} = 75 \text{ A}$$

12. a. $a = \frac{N_p}{N_s} = \frac{400 \text{ t}}{1200 \text{ t}} = \frac{1}{3}$

$$Z_i = a^2 Z_L = \left[\frac{1}{3} \right]^2 [9 \Omega + j12 \Omega] = 1 \Omega + j1.333 \Omega = 1.667 \Omega \angle 53.13^\circ$$

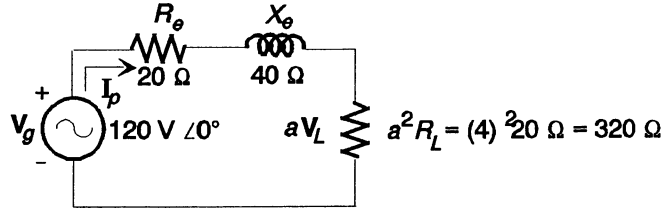
$$I_p = V_g / Z_i = 100 \text{ V} / 1.667 \Omega = 60 \text{ A}$$

b. $I_L = aI_p = \frac{1}{3}(60 \text{ A}) = 20 \text{ A}$, $V_L = I_L Z_L = (20 \text{ A})(15 \Omega) = 300 \text{ V}$

14. a. $R_e = R_p + a^2 R_s = 4 \Omega + (4)^2 1 \Omega = 20 \Omega$

b. $X_e = X_p + a^2 X_s = 8 \Omega + (4)^2 2 \Omega = 40 \Omega$

c.



d. $I_p = \frac{V_g}{Z_p} = \frac{120 \text{ V } \angle 0^\circ}{20 \Omega + 320 \Omega + j40 \Omega} = \frac{120 \text{ V } \angle 0^\circ}{340 \Omega + j40 \Omega} = 0.351 \text{ A } \angle -6.71^\circ$

e. $aV_L = \frac{a^2 R_L V_g}{(R_e + a^2 R_L) + jX_e} = I_p a^2 R_L$
or $V_L = aI_p R_L \angle 0^\circ = (4)(0.351 \text{ A } \angle -6.71^\circ)(20 \Omega \angle 0^\circ) = 28.1 \text{ V } \angle -6.71^\circ$

f. —

g. $V_L = \frac{N_s}{N_p} V_g = \frac{1}{4}(120 \text{ V}) = 30 \text{ V}$

16. a. $a = N_p/N_s = 4 \text{ t}/1 \text{ t} = 4$, $R_e = R_p + a^2 R_s = 4 \Omega + (4)^2 1 \Omega = 20 \Omega$
 $X_e = X_p + a^2 X_s = 8 \Omega + (4)^2 2 \Omega = 40 \Omega$
 $Z_p = R_e + jX_e - ja^2 X_C = 20 \Omega + j40 \Omega - j(4)^2 20 \Omega$
 $= 20 \Omega - j280 \Omega = 280.71 \Omega \angle -85.91^\circ$

b. $I_p = \frac{V_g}{Z_p} = \frac{120 \text{ V } \angle 0^\circ}{280.71 \Omega \angle -85.91^\circ} = 0.427 \text{ A } \angle 85.91^\circ$

c. $V_{R_e} = (I_p \angle \theta)(R_e \angle 0^\circ) = (0.427 \text{ A } \angle 85.91^\circ)(20 \Omega \angle 0^\circ) = 8.54 \text{ V } \angle 85.91^\circ$
 $V_{X_e} = (I_p \angle \theta)(X_e \angle 90^\circ) = (0.427 \text{ A } \angle 85.91^\circ)(40 \Omega \angle 90^\circ) = 17.08 \text{ V } \angle 175.91^\circ$
 $V_{X_C} = (I_p \angle \theta)(a^2 X_C \angle -90^\circ) = (0.427 \text{ A } \angle 85.91^\circ)(320 \Omega \angle -90^\circ) = 136.64 \text{ V } \angle -4.09^\circ$

18. Coil 1: $L_1 - M_{12}$
Coil 2: $L_2 - M_{12}$
 $L_T = L_1 + L_2 - 2M_{12} = 4 \text{ H} + 7 \text{ H} - 2(1 \text{ H}) = 9 \text{ H}$

20. $M_{23} = k\sqrt{L_2 L_3} = 1\sqrt{(1 \text{ H})(4 \text{ H})} = 2 \text{ H}$
Coil 1: $L_1 + M_{12} - M_{13} = 2 \text{ H} + 0.2 \text{ H} - 0.1 \text{ H} = 2.1 \text{ H}$
Coil 2: $L_2 + M_{12} - M_{23} = 1 \text{ H} + 0.2 \text{ H} - 2 \text{ H} = -0.8 \text{ H}$
Coil 3: $L_3 - M_{23} - M_{13} = 4 \text{ H} - 2 \text{ H} - 0.1 \text{ H} = 1.9 \text{ H}$
 $L_T = 2.1 \text{ H} - 0.8 \text{ H} + 1.9 \text{ H} = 3.2 \text{ H}$

$$\begin{aligned}
22. \quad \mathbf{Z}_i &= \mathbf{Z}_p + \frac{(\omega M)^2}{\mathbf{Z}_s + \mathbf{Z}_L} = R_p + jX_{L_p} + \frac{(\omega M)^2}{R_s + jX_{L_s} + R_L} \\
R_p &= 2 \, \Omega, \quad X_{L_p} = \omega L_p = (10^3 \text{ rad/s})(8 \text{ H}) = 8 \text{ k}\Omega \\
R_s &= 1 \, \Omega, \quad X_{L_s} = \omega L_s = (10^3 \text{ rad/s})(2 \text{ H}) = 2 \text{ k}\Omega \\
M &= k\sqrt{L_p L_s} = 0.05\sqrt{(8 \text{ H})(2 \text{ H})} = 0.2 \text{ H} \\
\mathbf{Z}_i &= 2 \, \Omega + j8 \text{ k}\Omega + \frac{(10^3 \text{ rad/s} \cdot 0.2 \text{ H})^2}{1 \, \Omega + j2 \text{ k}\Omega + 20 \, \Omega} \\
&= 2 \, \Omega + j8 \text{ k}\Omega + \frac{4 \times 10^4 \, \Omega}{21 + j2 \times 10^3} \\
&= 2 \, \Omega + j8 \text{ k}\Omega + 0.21 \, \Omega - j19.99 \, \Omega = 2.21 \, \Omega + j7980 \, \Omega \\
\mathbf{Z}_i &= 7980 \, \Omega \angle 89.98^\circ
\end{aligned}$$

$$\begin{aligned}
24. \quad I_s &= I_1 = 2 \text{ A}, \quad E_p = V_L = 40 \text{ V} \\
V_g I_1 &= V_L I_L \Rightarrow I_L = V_g / V_L \cdot I_1 = \frac{200 \text{ V}}{40 \text{ V}} (2 \text{ A}) = 10 \text{ A} \\
I_p + I_1 &= I_L \Rightarrow I_p = I_L - I_1 = 10 \text{ A} - 2 \text{ A} = 8 \text{ A}
\end{aligned}$$

$$\begin{aligned}
26. \quad \text{a.} \quad \mathbf{E}_2 &= \frac{N_2}{N_1} \mathbf{E}_1 = \frac{15 \text{ t}}{90 \text{ t}} (60 \text{ V} \angle 0^\circ) = 10 \text{ V} \angle 0^\circ \\
\mathbf{E}_3 &= \frac{N_3}{N_1} \mathbf{E}_1 = \frac{45 \text{ t}}{90 \text{ t}} (60 \text{ V} \angle 0^\circ) = 30 \text{ V} \angle 0^\circ \\
\mathbf{I}_2 &= \frac{\mathbf{E}_2}{\mathbf{Z}_2} = \frac{10 \text{ V} \angle 0^\circ}{8 \, \Omega \angle 0^\circ} = 1.25 \text{ A} \angle 0^\circ \\
\mathbf{I}_3 &= \frac{\mathbf{E}_3}{\mathbf{Z}_3} = \frac{30 \text{ V} \angle 0^\circ}{5 \, \Omega \angle 0^\circ} = 6 \text{ A} \angle 0^\circ
\end{aligned}$$

$$\begin{aligned}
\text{b.} \quad \frac{1}{R_1} &= \frac{1}{(N_1/N_2)^2 R_2} + \frac{1}{(N_1/N_3)^2 R_3} \\
&= \frac{1}{(90 \text{ t}/15 \text{ t})^2 8 \, \Omega} + \frac{1}{(90 \text{ t}/45 \text{ t})^2 5 \, \Omega} \\
\frac{1}{R_1} &= \frac{1}{288 \, \Omega} + \frac{1}{20 \, \Omega} = 0.05347 \text{ S} \\
R_1 &= 18.70 \, \Omega
\end{aligned}$$

$$28. \quad \mathbf{Z}_M = \mathbf{Z}_{M_{12}} = \omega M_{12} \angle 90^\circ$$

$$\begin{aligned}
&\mathbf{E} - \mathbf{I}_1 \mathbf{Z}_1 - \mathbf{I}_1 \mathbf{Z}_{L_1} - \mathbf{I}_1 (-\mathbf{Z}_m) - \mathbf{I}_2 (+\mathbf{Z}_m) - \mathbf{I}_1 \mathbf{Z}_{L_2} + \mathbf{I}_2 \mathbf{Z}_{L_2} - \mathbf{I}_1 (-\mathbf{Z}_m) = 0 \\
&\mathbf{E} - \mathbf{I}_1 (\mathbf{Z}_1 + \mathbf{Z}_{L_1} - \mathbf{Z}_m + \mathbf{Z}_{L_2} - \mathbf{Z}_m) - \mathbf{I}_2 (\mathbf{Z}_m - \mathbf{Z}_{L_2}) = 0 \\
\text{or} \quad &\mathbf{I}_1 (\mathbf{Z}_1 + \mathbf{Z}_{L_1} + \mathbf{Z}_{L_2} - 2 \mathbf{Z}_m) + \mathbf{I}_2 (\mathbf{Z}_m - \mathbf{Z}_{L_2}) = \mathbf{E} \\
\hline
&-\mathbf{I}_2 \mathbf{Z}_2 - \mathbf{Z}_{L_2} (\mathbf{I}_2 - \mathbf{I}_1) - \mathbf{I}_1 (+\mathbf{Z}_m) = 0 \\
\text{or} \quad &\mathbf{I}_1 (\mathbf{Z}_m - \mathbf{Z}_{L_2}) + \mathbf{I}_2 (\mathbf{Z}_2 + \mathbf{Z}_{L_2}) = 0 \\
\hline
\end{aligned}$$